

# Optimized multi-junction photovoltaic solar cells for terrestrial applications

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## Abstract

Sunlight, undoubtedly, is the future source of energy for humans. The current conversion technologies of sunlight to electricity using photovoltaic technologies are still moderate and relatively costly. The multi-junction photovoltaic cell design offers the highest achieved conversion efficiency that already exceeded the 40%. This work aims for maximizing the conversion of the sunlight for terrestrial application using the series-multi-junction photovoltaic cells by an optimal choice for the bandgap energies.

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**Keywords:** Solar energy conversion; Photovoltaic cells; Multi-junction; Bandgap energy; Cutoff wavelength; Optimized design

## 1. Introduction

Solar energy conversion to other forms of energy is essential for us. Nowadays, modern cities are run mainly by electricity; therefore, the conversion of the sunlight to electricity is the most appealing technology that utilizes the solar energy.

There are different known approaches for converting sunlight to electricity. The solar-thermal scheme that depends on producing pressurized steam yields about 15% conversion efficiency; such scheme is widely used because it can be employed at large scale solar farms with reasonable cost. Another similar solar-thermal approach is to use the concentrated sunlight to drive Stirling engine (Kongtragool and Wongwises, 2003; Exergy). The net efficiency of the last scheme is relatively high, about 35%; nevertheless, it has limited use because technically is hard to deploy at large scale and maintain reasonable cost.

The most promising technology for converting sunlight to electricity is the photovoltaic PV cell technology (IEA;

Solar Technologies Market Report, 2010), which depends on converting the photons flux in the sunlight directly to a direct electric current using the photovoltaic effect. Nevertheless, the PV cells manufacturing is a high-tech that requires a costly fabrication environment that is not strongly justified by the current conversion efficiencies which is about 10–20% (Solar Technologies Market Report, 2010; Wolf, 1960). Therefore, many methods were invented and attempted to reach higher efficiencies (Kraets, 2010; Reisfeld and Neuman, 1978; Reisfeld and Jorgensen, 1982; Nelson, 2003; Bitnar, 2003; NREL; Cotal and et al., 2009; Takamoto et al., 2009; Edmondson et al., 2005). The multi-junction approach yields the most promising efficiency among all PV technologies since it employs a stack of junctions with different, but properly chosen, bandgap energies to better match the solar spectrum with maximum utilization and minimum thermalization loss (Cotal and et al., 2009; Takamoto et al., 2009; Edmondson et al., 2005; Nelson, 2003). A conversion efficiency that exceeded 40% has been reported with three-junction PV cells (Cotal and et al., 2009). Expectedly, introducing more junctions into the PV cell should improve the efficiency (De Vos, 1980); nevertheless, such approach is

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technologically more demanding with relatively higher cost. Therefore, careful design aspects need be accounted in order to reach optimal multi-junction design with maximum efficiency and ease the efficiency/cost trade-off issue. De Vos (1980) not only sets the theoretical limits of multi-junction solar cells based on the detailed balance limit theory, but also provides the optimal bandgap energies that yields the maximum conversion efficiency for the multi-junction solar cells that are employed for free space applications (i.e. at AM0). However, to the best of our knowledge, there was no useful attempt that optimizes the design of the multi-junction PV cells for terrestrial applications at AM1. Therefore, the author is proposing an optimization of the bandgap energies of a given number of junctions for the terrestrial multi-junction PV cells; which, indeed, has much broader application than free space multi-junction PV cells.

The limited conversion efficiency of PV cells is attributed to different kinds of losses (Solar Technologies Market Report, 2010; Wolf, 1960). The most prominent losses are the thermalization loss which is due to producing hot electrons from shorter wavelengths photons such as the ultraviolet range of the solar spectrum; and the non absorbed photons with longer wavelength such as the infrared range of the solar spectrum. Usually, the hot electrons lose their excess energy by producing phonons which are lost eventually as heat in the cell. Whereas, the longer wavelength photons have inadequate energy to launch free electrons, therefore, they are not utilized in the photovoltaic conversion process. A trade-off scenario is encountered as one attempts to find the optimal bandgap energy of a single cell that compromises the two kinds of losses. As a matter of fact, it was found that the efficiency limits of an optimized single junction and multi junction PV cell at AM0 ranges from 30% for a single junction to 68% for infinite number of junctions, which indeed sets the theoretical limit for the efficiency of the PV cell illuminated by sunlight at AM0 (De Vos, 1980; Shockley and Queisser, 1961).

## 2. Theory

The problem is to find the optimal set of bandgap energies of the semiconductor materials that will be used as the active regions in the multi-junctions structure in order to optimize the cell performance with maximum electrical power delivery for terrestrial applications. The solar spectrum at AM1 (Fig. 1) is usually obtained from tabulated experimental data and is not described by a theoretical or empirical function as it was for the works of De Vos (1980) and Shockley and Queisser (1961), which handled the AM0 case, and considered accurately the solar spectrum radiating off a black body at 6000 K. This makes the problem rather not a straight forward task. The approach here depends on a careful development of a theoretical model that accounts for the major losses in the photovoltaic conversion of sunlight to electricity, and then

followed by numerical optimization with respect to the cut-off wavelengths of the junctions that associate the bandgap energies of the active regions ( $\lambda_{\text{cutoff}} E_{\text{bandgap}} = 1.24$ , where  $\lambda_{\text{cutoff}}$  is in micrometer and  $E_{\text{bandgap}}$  is in electron volt).

As discussed above the concept of multi-junction in PV cell deals with two major issues that comes from the spectral mismatch between the solar spectrum and the conversion response of the photovoltaic junction. The first limiting effect comes from the thermalization losses which is due to the generation of hot electron by the absorption of photons with higher energy than the bandgap energy of the absorbing medium. Not only this excess energy is lost, but worse than that it is converted to internal heat that usually increases the cell's temperature, thus, lowers the performance and the life span of the cell because of the increased thermal stress inside the cell. The second limiting factor is that those low energy photons that are lower than the bandgap energy are not absorbed to be converted to electricity; therefore, they are not utilized in the conversion process. As a matter of fact, the multi-junction approach was proposed in order to deal with those two issues effectively because it depends on segmenting the solar spectrum among a cascaded group of PV junctions, similar to controlling the numerical integration error by increasing the number of segments. Moreover, the more segmentation of the solar spectrum permits broader cover for the solar spectrum with less concern of increasing the thermalization losses. Basically, as a rule of thumb, in order to better control the overall thermalization, the entering junction are assigned to absorb the shorter wavelengths of the solar spectrum as sunlight enters the multi-junction PV cell, whereas, the later junctions are used to absorb the longer wavelengths of the spectrum.

The next concern is to assign the cutoff wavelengths, which associate the bandgap energies, of the absorbing media among the solar spectrum to optimally control the overall thermalization losses and at the same time utilize as much as possible the far tail of the solar spectrum.

To start modeling this problem we assume that the number of junctions is given and the main task is to find the optimal bandgap energies that maximize the conversion efficiency; therefore, the thermalization losses in each junction is expressed by:

$$\begin{aligned} Th_k &= \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) (E_{ph}(\lambda) - E_{g,k}(\lambda_k)) d\lambda \\ &= \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) \left( \frac{hc}{\lambda} - \frac{hc}{\lambda_k} \right) d\lambda \end{aligned} \quad (1)$$

where  $\phi(\lambda)$  is the photon flux density of the sunlight,  $E_{ph}(\lambda)$  is the photon energy with wavelength  $\lambda$ ,  $E_{g,k}(\lambda_k)$  is the bandgap energy for the  $k$ th junction and  $\lambda_k$  is the corresponding cutoff wavelength,  $h$  is Planck's constant,  $c$  is the speed of light in vacuum.

Therefore, the total thermalization losses for  $n$ -junction PV cell with cutoff wavelengths  $\lambda = [\lambda_1, \lambda_2 \dots \lambda_k \dots \lambda_n]$  could be expressed as:

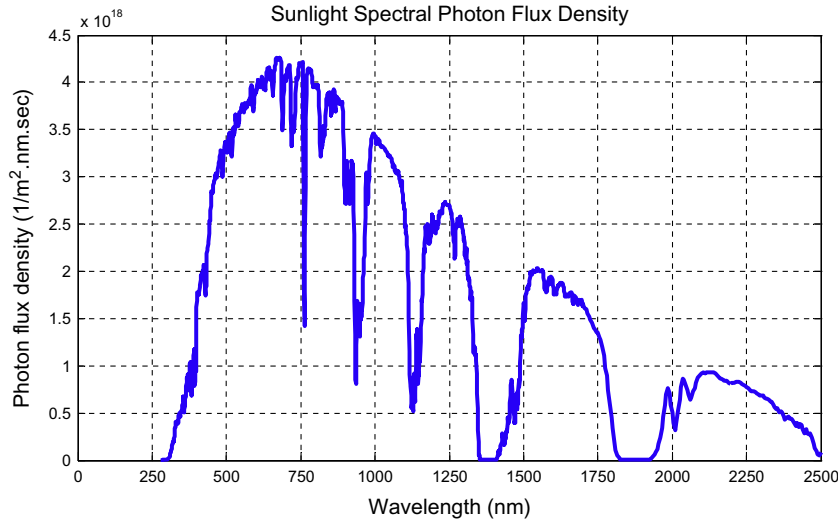


Fig. 1. Photon spectral density of sunlight at AM1.

$$J(A) = Th_{Tot} = \sum_{k=1}^n \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) \left( \frac{hc}{\lambda} - \frac{hc}{\lambda_k} \right) d\lambda \right) \\ = \int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda - hc \sum_{k=1}^n \frac{1}{\lambda_k} \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \right) \quad (2)$$

where  $\lambda_0$  is the shortest wavelength in the solar spectrum,  $p(\lambda)$  is sunlight power spectral density and is related to the photon flux density by  $p(\lambda) = \frac{hc}{\lambda} \phi(\lambda)$ .

In order to minimize the total thermalization losses, then partial derivation with respect to all cutoff wavelength must be set to zero. It worth noting that the  $J$  function has  $(n+1)$  terms, and the partial derivation with respect to  $\lambda_k$  (for  $k=1,2,3,\dots,(n-1)$ ) affects only the  $k$ -th and  $(k+1)$ -th; whereas, partial derivation with respect to  $\lambda_n$  affects the first and the last terms only; therefore, this leads to:

$$\frac{\partial J}{\partial \lambda_k} = 0 \Rightarrow \frac{\partial}{\partial \lambda_k} \left( \sum_{k=1}^n \frac{1}{\lambda_k} \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \right) \right) \\ = \frac{\partial}{\partial \lambda_k} \left( \left( \frac{1}{\lambda_k} \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \right) + \left( \frac{1}{\lambda_{k+1}} \int_{\lambda_k}^{\lambda_{k+1}} \phi(\lambda) d\lambda \right) \right) \\ = 0$$

For  $k=1,2,3,\dots,(n-1)$ ; and for  $k=n$  yields:

$$\frac{\partial J}{\partial \lambda_n} = 0 \Rightarrow \frac{\partial}{\partial \lambda_n} \left( \int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda \right) - \frac{\partial}{\partial \lambda_n} \left( \frac{hc}{\lambda_n} \int_{\lambda_{n-1}}^{\lambda_n} \phi(\lambda) d\lambda \right) = 0$$

Working out the two partial derivations and arranging yields the following conditions:

$$\phi(\lambda_k) \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_{k+1}} \right) = \frac{1}{\lambda_k^2} \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \\ \text{for } k=1,2,\dots,(n-2), \quad \text{and } \int_{\lambda_{n-1}}^{\lambda_n} \phi(\lambda) d\lambda = 0.$$

Which are reduced to:

$$\int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda = \phi(\lambda_k) (\lambda_{k+1} - \lambda_k) \left( \frac{\lambda_k}{\lambda_{k+1}} \right) \quad (3)$$

for  $k=1,2,\dots,(n-1)$ , and

$$\int_{\lambda_{n-1}}^{\lambda_n} \phi(\lambda) d\lambda = 0 \quad (4)$$

The last condition in Eq. (4) means that there are zero photons in a spectral width  $(\lambda_{n-1}, \lambda_n)$ , which leads with the account of Eq. (3) to a meaningless solution:  $(\lambda_0 = \lambda_1 = \lambda_2 = \dots = \lambda_n)$ .

In order to finish the optimization properly and reach useful results, we try to analyze how the merit function  $J$  behaves as the cutoff wavelengths changes. Apparently, the function  $J$  goes through a minimum with increasing each of the cutoff wavelengths for the first  $(n-1)$  junctions  $(\lambda_1, \lambda_2, \dots, \lambda_{n-1})$ , whereas  $J$  goes through a maximum as  $\lambda_n$  increases; which indeed presents a saddle point like. Therefore, to reach the optimal multi-junction PV cell design the set of conditions in Eq. (3) are accounted, whereas Eq. (4) is disregarded. In order to find the optimal set of cutoff wavelengths another major practical concern need be addressed here; which is the current matching among the junctions. It is well know that the electric circuit model of the PV cells is basically a network of a current source with series and parallel resistors and a diode; therefore, for those series connected current sources to deliver maximum electricity, then all currents should be matched, or at least close in value to each other within a tolerated differences. Moreover, assuming that the optical quantum efficiency, which is decided by the sunlight absorption coefficient and the thickness of the active region, of all junctions is close to unity because of choosing adequate thicknesses for the absorbing layers that also compromises the diffusion lengths in order to permit the photo-generated charges reach the electrodes, then the current that is produced from the junction  $k$  is proportional to the integration

of the sunlight photon flux density function over the spectral range  $[\lambda_k, \lambda_{k+1}]$ ; that is

$$\int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda = \alpha \quad (5)$$

where  $\alpha$  is a constant that equals the total number of photons in the utilized sunlight per sec per square meter divided by the number of used junctions, that is:

$$\int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda = \frac{1}{n} \int_{\lambda_0}^{\lambda_n} \phi(\lambda) d\lambda \text{ for } k = 1, 2, \dots, n. \quad (6)$$

Intuitively, the losses that are due to the current mismatch among the junctions have a ceiling value that is given by:

$$CM_{Tot} \leq \sum_{k=1}^n |I_k - I_{\min}| V_{oc,k} \quad (7)$$

where  $I_{\min} = \min(I_1, I_2, \dots, I_n)$ ,  $I_k \approx q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda$  is the photo-current produced from junction  $k$  (assuming unity for the optical and electrical quantum efficiencies),  $q$  is the elementary charge, and  $V_{oc,k}$  is the open circuit voltage for junction  $k$  which is obtained from experimental findings for a wide range of bandgap energies as (King et al., 2010)

$$V_{oc,k} \approx \frac{hc}{q\lambda_k} - 0.414 \quad (8)$$

Apparently, designing for the cutoff wavelengths  $[\lambda_1, \lambda_2, \dots, \lambda_n]$  which satisfy the two sets of conditions in Eqs. (3) and (6) is the basic requirement for minimum thermalization losses and current matching, respectively. Nevertheless, in order to obtain maximum electricity from the multi-junction structures the effective fill factor  $FF_{eff}$  need be accounted, which is another major effect that reduces the conversion efficiency and comes from the nonlinear nature of the  $I$ - $V$  characteristics of the illuminated junction and is given by:

$$I_k = I_{o,k} \left( \exp \left( \frac{V_k}{V_T} \right) - 1 \right) - I_{ph,k} \quad (9)$$

where  $I_k$  and  $V_k$  are the junction  $k$  operating current and voltage, respectively;  $I_{o,k}$  is the reverse saturation current of the junction  $k$ ,  $V_T = \frac{q}{kT} = 26$  mV at room temperature, and  $I_{ph,k}$  is the photo generated current of the junction  $k$  which practically equals the short circuit current  $I_{sc,k}$  that is  $I_{ph,k} \approx I_{sc,k}$ , and for unity quantum efficiency is expressed by:

$$I_{ph,k} \approx q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \quad (10)$$

Eq. (9) can be rewritten in terms of the open circuit voltage  $V_{oc,k}$  and the photo generated current  $I_{ph,k}$  as:

$$V_k \approx V_{oc,k} + V_T \ln \left( \frac{I_k}{I_{ph,k}} + 1 \right) \quad (11)$$

Substituting Eqs. (8) and (10) in Eq. (11) yield:

$$V_k \approx \frac{1240}{\lambda_k} + 0.026 \ln \left( \frac{I_k}{q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda} + 1 \right) - 0.414 \quad (12)$$

where  $\lambda_k$  is in nm and room the temperature was assumed.

Therefore, the total electric power obtained from the multi-junction structure can be expressed by:

$$\begin{aligned} P_{elect} &= \sum_{k=1}^n I_k V_k \\ &= \sum_{k=1}^n I_k \left[ \frac{1240}{\lambda_k} + 0.026 \ln \left( \frac{I_k}{q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda} + 1 \right) - 0.414 \right] \end{aligned} \quad (13)$$

Hence, the net efficiency is expressed by:

$$\eta = \frac{\sum_{k=1}^n I_k \left[ \frac{1240}{\lambda_k} + 0.026 \ln \left( \frac{I_k}{q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda} + 1 \right) - 0.414 \right]}{\int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda} \quad (14)$$

where the integration in the dominator of Eq. (14) is found at AM1 as 893.9 W/m<sup>2</sup>.

But since the junctions are connected in series, they should have one current  $I_{cell}$  with upper limit as  $I_{\min} = \min(I_1, I_2, \dots, I_n)$ , therefore Eq. (14) becomes:

$$\eta = \frac{\sum_{k=1}^n I_{cell} \left[ \frac{1240}{\lambda_k} + 0.026 \ln \left( \frac{I_{cell}}{q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda} + 1 \right) - 0.414 \right]}{\int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda} \quad (15)$$

Moreover, in order to find the  $FF_{eff}$ , the efficiency can be expressed differently by:

$$\begin{aligned} \eta &= \frac{\int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda - Th_{Tot}}{\int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda} FF_{eff} \\ &= \frac{hc \sum_{k=1}^n \frac{1}{\lambda_k} \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \right)}{\int_{\lambda_0}^{\lambda_n} p(\lambda) d\lambda} FF_{eff} \end{aligned} \quad (16)$$

where  $FF_{eff}$  is the effective fill factor of the whole structure, which is found by substituting Eq. (15) in Eq. (16) and rearranging as:

$$FF_{eff} = \frac{\sum_{k=1}^n I_{cell} \left[ \frac{1240}{\lambda_k} + 0.026 \ln \left( \frac{I_{cell}}{q \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda} + 1 \right) - 0.414 \right]}{hc \sum_{k=1}^n \frac{1}{\lambda_k} \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda \right)} \quad (17)$$

Obviously, numerical solution for this problem is our choice here as will be discussed in the following section.

It is important to mention here that the major factors which limit the photovoltaic conversion efficiency in the multijunction PV cells have been accounted here; nevertheless, the actual efficiency should be less than the efficiency

obtained from Eq. (15), since some unavoidable but less limiting factors (besides the assumed unity factors for the optical and the quantum efficiencies) like the shading losses of the electrode metallization, and the reflection of light at the air-cell interface and at the interfaces among the layers inside the multi-junction structure which practically never be zero but could be controlled effectively by using anti reflective coatings.

### 3. Analysis and results

The search for the cutoff wavelengths that produces the optimal conversion efficiency will be based on a random search within a limited, but proper, spectral bands as will be discussed below. It starts by finding those cutoff wavelengths that permit minimum thermalization losses in multi-junction PV cell and satisfy Eq. (3); from which the photo-generated currents of all junctions can be determined using Eq. (10). Consequently, the ceiling min ( $I_1, I_2, \dots, I_n$ ) of the cell current  $I_{cell}$  can be determined. Then the cell current is allowed to vary below its ceiling value and the resulted maximum conversion efficiency is estimated using Eq. (15) and temporally saved with the corresponding cutoff wavelengths. Iteratively, another random guess within the allowed spectral bands is taken and the resulted maximum conversion efficiency is estimated as before; only that efficiency estimate that score higher is recorded with its corresponding cutoff wavelengths; therefore, an evolutionary history of the best guesses for the cutoff wavelengths is obtained.

In order to find a proper initial guesses of the cutoff wavelengths to start the random search for the optimal cutoff wavelengths one need to investigate the peculiarities of the photon flux density spectra of the sunlight that is shown in Fig. 1. Because of the atmospheric interaction with sunlight

as many as eleven peculiarities could be detected through this spectra; some of those peculiarities appear as sharp dips, while others, like those comes in the infra red region, appear as wavelength bands of total absorption for sunlight. Table 1 summarize those spectral peculiarities with the corresponding center wavelength location of that spectral peculiarity, spectral width, photon density change, and accumulative photon count from the starting wavelength of sunlight spectra  $\lambda_0 = 280$  nm to the center wavelength of the spectral peculiarity. It would be helpful that if some of the initial guesses that are used for searching the optimal set of cutoff wavelengths is taken at any of those peculiarities since the dip spectral peculiarities helps to tune the photon flux density without practically changing the wavelength, whereas, those band peculiarities do the opposite. Such decoupling between the photon flux density and the wavelength at the spectral peculiarities once used as initial guesses makes the search much reliable and fast.

All the sunlight spectral peculiarities, which shall be used as reference wavelengths for searching the optimal design, comes after the wavelength that correspond to maximum photon flux density  $\lambda_{max} = 675$  nm; therefore, Table 2 was produced to describe the accumulative photon in the spectral region, which also helps deciding an initial guess for the first cutoff wavelength(s).

Now depending on the observations above, one can start an effective and simple search method for locating the optimal design. After choosing the proper initial guesses of the cutoff wavelengths that are favored to be at the center wavelength of the properly chosen spectral peculiarities, and by permitting iterative random guesses within a reasonable spectral width one could find the optimal set of cutoff wavelengths with affordable number of iterations. In order to make the random search practical and effective, we define a monitoring value that helps

Table 1  
Summary of the spectral peculiarities due to atmospheric interaction in the solar spectrum.

| $\lambda$ (nm) | $\phi(\lambda) \times 10^5$ ( $\text{m}^{-2} \text{nm}^{-1} \text{s}^{-1}$ ) | $\Delta\lambda$ (nm) | $\Delta\phi$ ( $\text{m}^{-2} \text{nm}^{-1} \text{s}^{-1}$ ) | $\int_{\lambda_0}^{\lambda} \phi(\lambda) d\lambda \times 10^{21}$ ( $\text{m}^{-2} \text{s}^{-1}$ ) |
|----------------|------------------------------------------------------------------------------|----------------------|---------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| 689            | 5.693                                                                        | 10                   | 1.156                                                         | 0.9513                                                                                               |
| 720            | 5.573                                                                        | 17                   | 1.149                                                         | 1.0635                                                                                               |
| 762            | 2.306                                                                        | 7                    | 4.514                                                         | 1.2080                                                                                               |
| 819            | 5.433                                                                        | 21                   | 1.003                                                         | 1.4055                                                                                               |
| 936            | 1.313                                                                        | 37                   | 3.905                                                         | 1.7623                                                                                               |
| 1124           | 1.003                                                                        | 44                   | 3.465                                                         | 2.2286                                                                                               |
| 1269           | 3.529                                                                        | 12                   | 0.700                                                         | 2.5126                                                                                               |
| 1384           | 0.00                                                                         | 148                  | 3.228                                                         | 2.6497                                                                                               |
| 1880           | 0.00                                                                         | 165                  | 1.237                                                         | 3.1523                                                                                               |
| 2010           | 0.524                                                                        | 30                   | 0.732                                                         | 3.1802                                                                                               |
| 2065           | 3.529                                                                        | 12                   | 0.340                                                         | 3.2149                                                                                               |
| 2500           | 0.00                                                                         | —                    | —                                                             | 3.4667                                                                                               |

Table 2  
Samples of total solar photons counts in a spectral interval [240 nm,  $\lambda$ ].

| $\lambda$ (nm)                                                                                                            | 325  | 350  | 375  | 400  | 425  | 450  | 475  | 500  | 525  | 550  | 575  | 600  | 625  | 650  | 675  |
|---------------------------------------------------------------------------------------------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $\int_{\lambda_0}^{\lambda} \phi(\lambda) d\lambda \times 10^{20}$ ( $\text{m}^{-2} \text{s}^{-1}$ ) $\lambda_0 = 280$ nm | 0.02 | 0.12 | 0.28 | 0.51 | 0.95 | 1.47 | 2.15 | 2.89 | 3.66 | 4.48 | 5.34 | 6.22 | 7.13 | 8.05 | 8.99 |



deciding if the current random guess could do better than all previous guesses as:

$$\sigma = \left( \sum_{k=1}^{n-1} \left( \int_{\lambda_{k-1}}^{\lambda_k} \phi(\lambda) d\lambda - \phi(\lambda_k)(\lambda_{k+1} - \lambda_k) \left( \frac{\lambda_k}{\lambda_{k+1}} \right) \right)^2 \right)^{1/2} \quad (18)$$

Apparently, the smaller is the monitoring measure  $\sigma$  the better Eq. (3) is satisfied and the more likely that the current initial guess would leads to higher efficiency than the previous random guesses.

Equipped with the above guiding theory and the simple, but effective, numerical random searching tool we consider

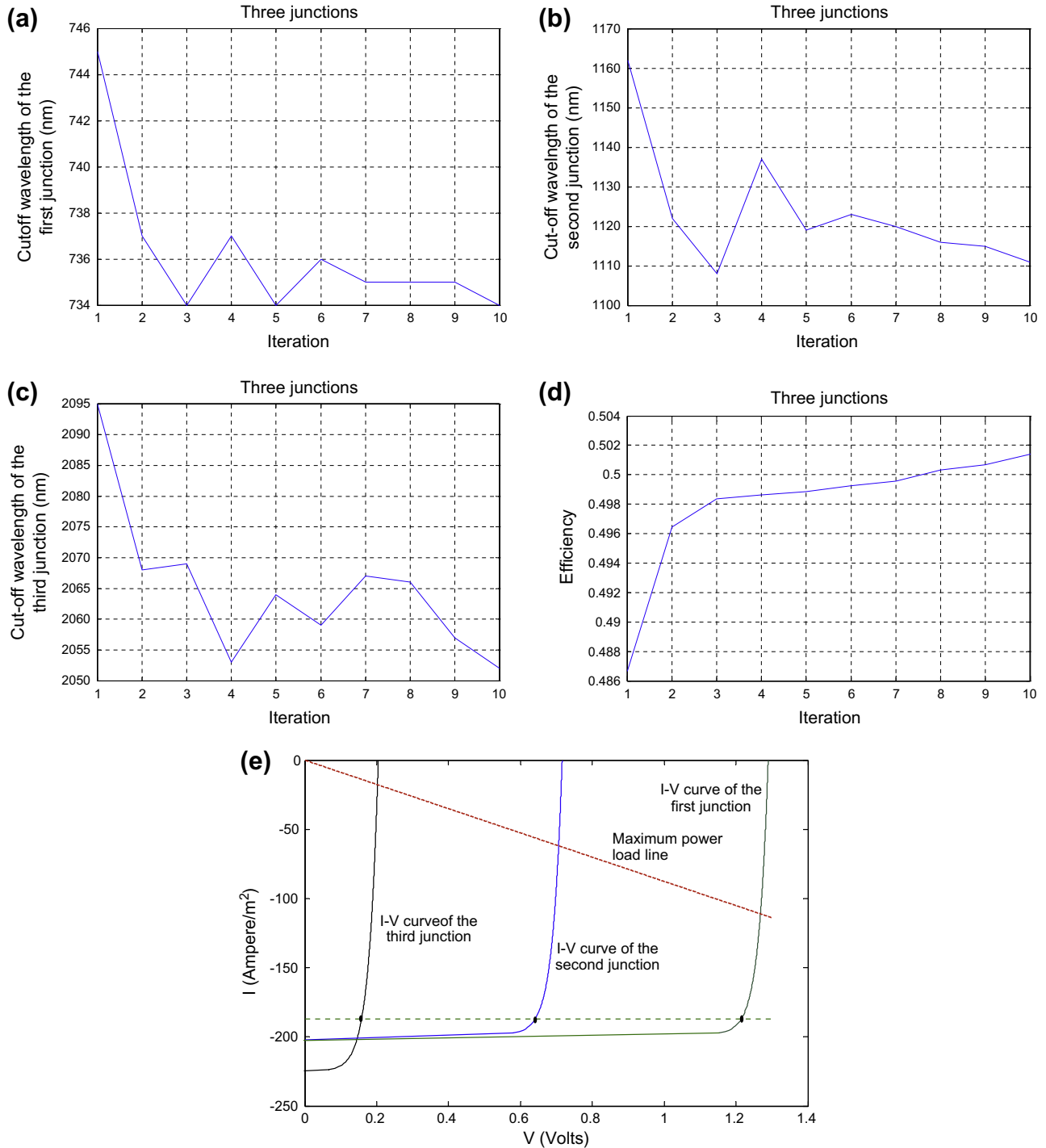


Fig. 2. (a) Evolution of the optimal cutoff wavelength search for the first junction in a three-junction PV cell, (b) evolution of the optimal cutoff wavelength search for the second junction in a three-junction PV cell, (c) evolution of the optimal cutoff wavelength search for the third junction in a three-junction PV cell, (d) evolution of the conversion efficiency during the search for the optimal three-junction PV cell, and (e)  $I$ - $V$  curves for the three junctions of the optimal three-junction PV cell at AM1 solar illumination. Solid dots on the curves are the maximum power operating points. The dashed sloped line is the maximum power load line.

below three examples for multi-junction PV cell optimal design.

The first example aims to optimally design a three-junction PV cell by choosing the cutoff wavelength for each junction in order to obtain the optimal utilization of the solar spectrum and maximum conversion of sunlight to electricity. Before choosing the proper initial guesses for the three cutoff wavelengths the value of  $\alpha$  was estimated from Eq. (5) as  $1.16 \times 10^{21}$  ( $1/\text{m}^2 \text{ s}$ ). Examining Tables 1 and 2 one finds the nearest wavelength peculiarity is 762 nm, which shall be the initial guess for the cutoff wavelength of the first junction. Then we try to find the second wavelength peculiarity that associates an accumulative

photon count as  $2.32 \times 10^{21}$  ( $1/\text{m}^2 \text{ s}$ ); again examining Table 2 leads to choosing 1124 nm as the initial guess for the cutoff wavelength of the second junction. The initial guess for the third junction is taken at 2200 nm which coincides near the last peak of the solar spectrum in order to account most of the solar power in the photovoltaic conversion. The three initial guesses for the cutoff wavelengths (762 nm, 1124 nm, 2200 nm) were loaded to a Matlab code that implements the random search method that is described above. Results are shown in Fig. 2; Fig. 2(a–c) shows the evolution history of the best random guesses of all three cutoff wavelengths that associate the first, second, and third junction, respectively. Fig. 2(d) shows the

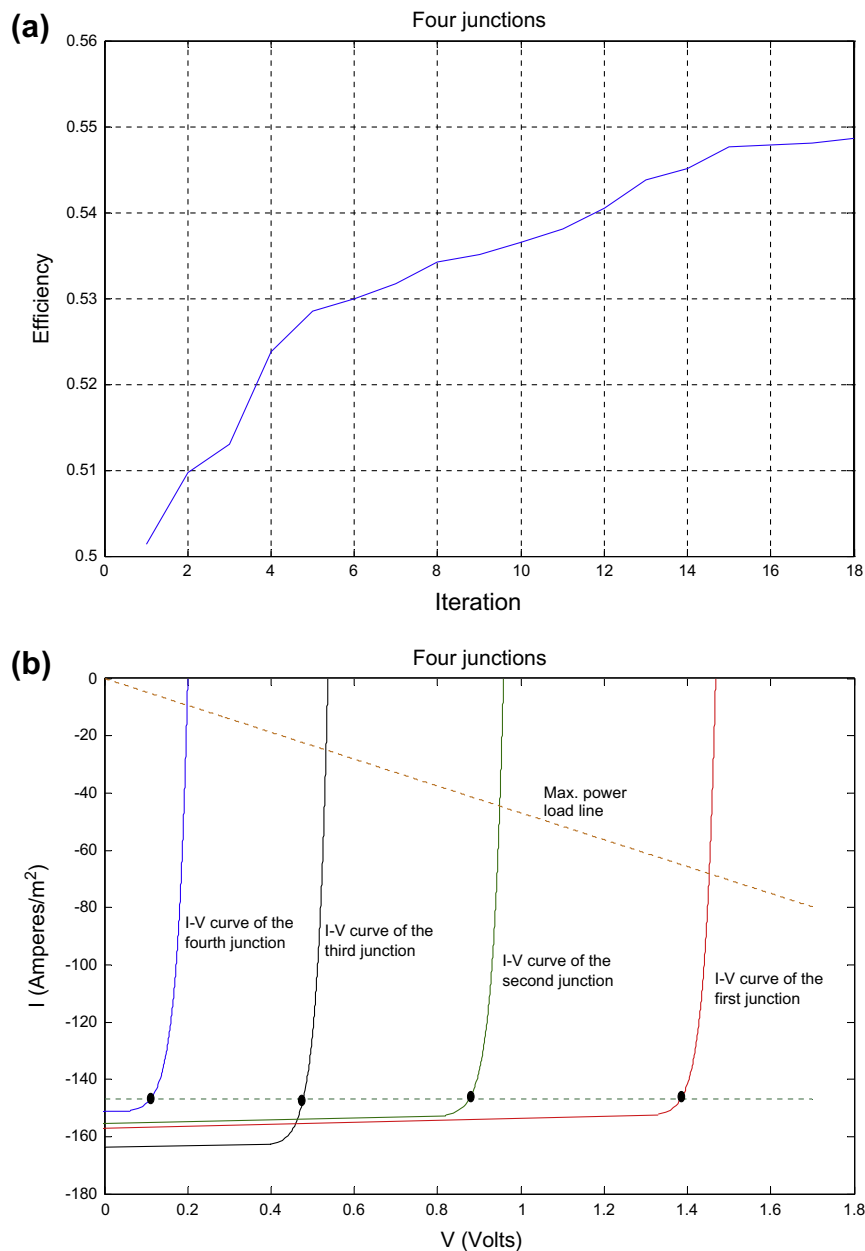


Fig. 3. (a) Evolution of the conversion efficiency during the search for the optimal four-junction PV cell and (b)  $I$ – $V$  curves for the three junctions of the optimal four-junction PV cell at AM1 solar illumination. Solid dots on the curves are the maximum power operating points. The dashed sloped line is the maximum power load line.

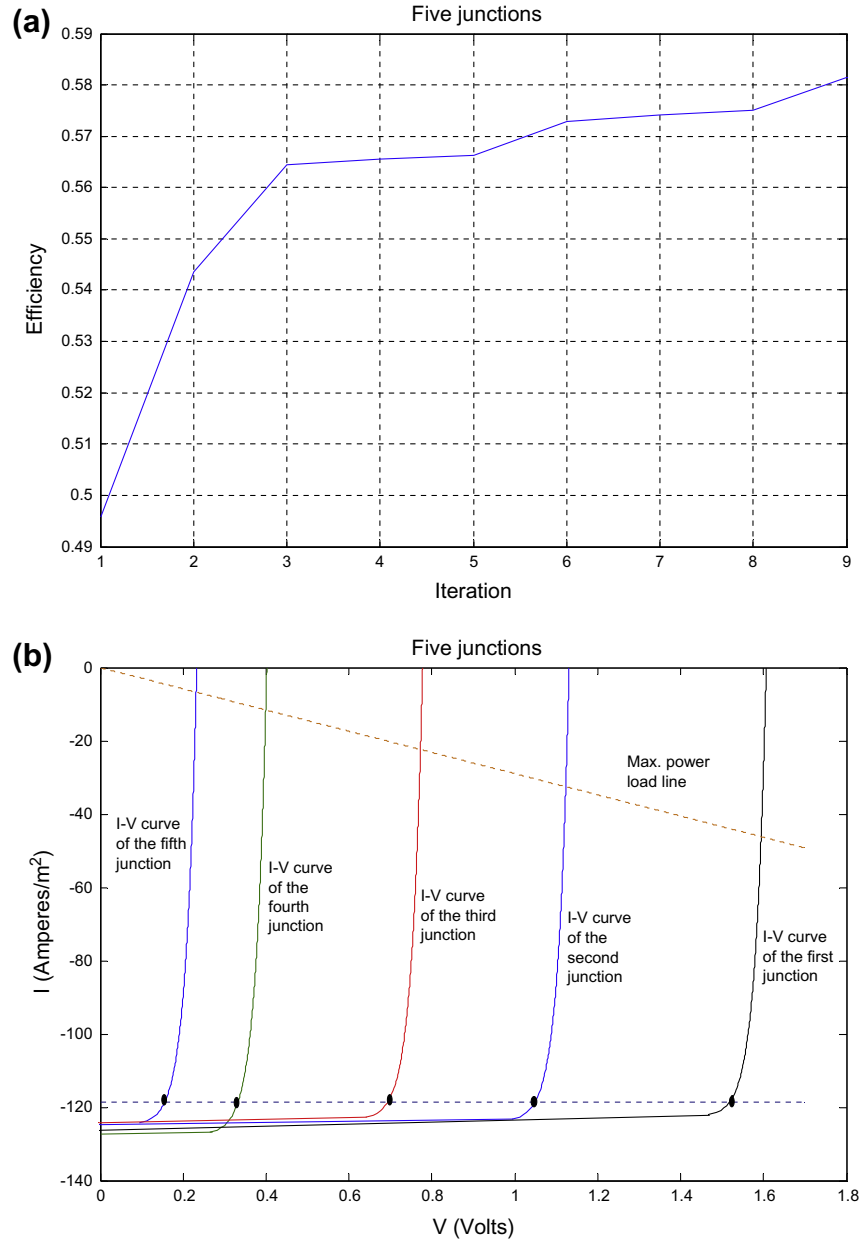


Fig. 4. (a) Evolution of the conversion efficiency during the search for the optimal five-junction PV cell and (b)  $I$ - $V$  curves for the three junctions of the optimal five-junction PV cell at AM1 solar illumination. Solid dots on the curves are the maximum power operating points. The dashed sloped line is the maximum power load line.

Table 3

Summary of the initial guesses and the reached optimized cutoff wavelengths for the three-, four- and five-junction PV cell.

| Cutoff wavelength (nm) | Three-junction PV cell |                 | Four-junction PV cell |                 | Five-junction PV cell |                 |
|------------------------|------------------------|-----------------|-----------------------|-----------------|-----------------------|-----------------|
|                        | Initial guess          | Final optimized | Initial guess         | Final optimized | Initial guess         | Final optimized |
| $\lambda_1$            | 762                    | 735             | 670                   | 664             | 620                   | 618             |
| $\lambda_2$            | 1124                   | 1115            | 936                   | 913             | 820                   | 810             |
| $\lambda_3$            | 2200                   | 2050            | 1384                  | 1324            | 1100                  | 1053            |
| $\lambda_4$            |                        |                 | 2200                  | 2066            | 1500                  | 1547            |
| $\lambda_5$            |                        |                 |                       |                 | 2100                  | 1960            |

corresponding evolution history for the conversion efficiency, and Fig. 2(e) shows the resulted  $I$ - $V$  curve of the three junctions with the maximum power load line as

a sloped dotted line, whereas, the maximum power point of each junction are shown on the three curves as solid dots.



Table 4

Summary of the thermalization losses, effective fill factor, total losses, and the corresponding conversion efficiency for the optimized three-, four- and five-junction PV cells.

|                                           | Three-junction PV cell | Four-junction PV cell | Five-junction PV cell |
|-------------------------------------------|------------------------|-----------------------|-----------------------|
| Thermalization losses (W/m <sup>2</sup> ) | 205.1                  | 156.1                 | 132.1                 |
| Effective Fill factor                     | 0.6503                 | 0.6646                | 0.6822                |
| Total losses (W/m <sup>2</sup> )          | 446.2                  | 403.6                 | 374.2                 |
| Conversion efficiency (%)                 | 50.1                   | 54.9                  | 58.1                  |

The problem solution was repeated for a four-junction PV cell to verify that efficiency would improve. Results are shown in Fig. 3; Fig. 3(a) shows the corresponding evolution history for the conversion efficiency, and Fig. 3(b) shows the resulted  $I$ – $V$  curve of the four junctions with the maximum power load line as a sloped dotted line, whereas, the maximum power point of each junction are shown on the four curves as solid dots. Finally, the problem solution was repeated for a five-junction PV cell in order to verify if further efficiency improvement can be obtained. Results are shown in Fig. 4. Fig. 4(a) shows the corresponding evolution history for the conversion efficiency, and Fig. 4(b) shows the resulted  $I$ – $V$  curve of the five junctions with the maximum power load line as a sloped dotted line, whereas, the maximum power point of each junction are shown on the five curves as solid dots.

Final results for the three examples were tabulated in Tables 3 and 4. Table 3 summarizes the initial guesses and the final optimized cutoff wavelengths for the three-junction, four-junction and five-junction PV cells. And Table 4 summarizes the obtained thermalization losses and the effective fill factor, the total losses, and the corresponding overall conversion efficiency for the three-junction, the four-junction and the five-junction PV cells.

The three designs, indeed, are showing new levels for conversion efficiencies. Expectedly, the highest and the most promising is 58.1% which comes from the optimized five-junction PV cell.

It worth mentioning here that the achieved conversion efficiencies is 1–3% higher than theoretical limit that was set by the detailed balance limit theory (De Vos, 1980). This can be attributed to the fact that the solar spectrum at AM1 was accounted here, unlike the detailed balance limit theory which accounts the free space solar spectrum as black-body radiating at 6000 K. Therefore, the reason for higher efficiency is not only because the solar spectrum at AM1 has less total power than the solar spectrum at AM0 (which are the normalized values in estimating the efficiency), but also it exhibits total absorption bands that could facilitate a cutoff wavelength switching between two adjacent junctions. Moreover, our optimized design, besides providing higher efficiencies and potentially have broader employment for the terrestrial applications, it depends on the series connection of junctions, therefore, holds a more practical design.

Finally, the founds of this work could be useful to choose materials properly; nevertheless, more work in material science is needed in order to deal with some fabri-

cation technical issues, like the lattice constant matching between the layers of the multi-junction structure; specially, if those devices are used in the concentrated sun regime in order to justify the higher cost. Furthermore, the optimally obtained efficient PV conversion is expected to have less heating of the cell, hence, provide more luxury for higher concentration and then cost justification.

#### 4. Summary

Optimal designs for multi-junction PV cell that depends on the more practical series connection of junctions for terrestrial applications was attempted in order to improve the conversion efficiency of the photovoltaic technology. The optimization was based on controlling the major losses that comes from the thermalization and the fill factor. Spectral peculiarities were detected and used as initial guess for the cutoff wavelengths, and then random search within a suitable spectral band was carried to reach the optimal set of the cutoff wavelengths that associate the junctions, from which the energy bandgap estimated and the materials choice could be defined. The three-junction, four-junction and five-junction PV cells optimal designs where investigated and expectedly the five-junction PV cell was found to outperform the other designs. The multi-junction PV cell is a promising technology to harvest the sunlight in the form of electricity; therefore, optimizing their designs to reach new boundaries of higher conversion efficiencies is useful for the human energy-demanding-progress.

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